Title	RELAXATION PROCESS IN A NONLINER SYSTEM WITH CHAOTIC FORCE(Session II : Chaos, The 1st Tohwa University International Meeting on Statistical Physics Theories, Experiments and Computer Simulations)
Author(s)	Morioka, Nozomi
Citation	物性研究 (1996), 66(3): 470-471
Issue Date	1996-06-20
URL	http://hdl.handle.net/2433/95806
Right	
Type	Departmental Bulletin Paper
Textversion	publisher

RELAXATION PROCESS IN A NONLINER SYSTEM WITH CHAOTIC FORCE

Nozomi Morioka
Department of Electrical Engineering, Kokushikan University, Tokyo 154, Japan

Since the pioneering works of Lorenz and Ueda, the difference between the usual random noise (for example,white-Gaussian noise) and the chaotic time behavior has been investigated by many authors in various fields. One of characteristic features of chaos is that the chaotic time behavior is deterministic. The stochastic nature comes from the sensitive dependence of the time behavior on the initial condition. The second one is its coherent nature, whose example may be seen in windows appearing in chaotic regions of the bifurcation parameter. That is, chaos has bilateral aspects.

We have previously studied the Brownian motion with a chaotic force instead of the usual random force,

$$\dot{x}(t) = -\gamma x(t) + f(t). \tag{1}$$

The chaotic force $f(t)$ changes chaotically at time intervals of τ,

$$f(t) = \frac{K}{\sqrt{\tau}}(y_{n+1} - < y_0 >) \ \text{ for } \ n\tau < t \leq (n+1)\tau \ \ (n = 0, 1, 2, \ldots), \tag{2}$$

where y_{n+1} is the (n+1)th iterate of a map F(y): $y_{n+1} = F(y_n)$.

In (2) K is the magnitude of the force and the factor $1/\sqrt{\tau}$ is needed to get a finite diffusion constant for the small τ limit (small but finite). We obteined that if τ is much larger than the decay time $\tau_d = 1/\gamma$; $\tau \gg \tau_d$, the stationary distribution $P(x)$ has the same form sa that of the invariant density of $F(y)$. On the other hand, for small $\tau(\gamma\tau \ll 1)$ we can derive the Fokker-Planck type equation with memory effects. If the correlation of the chaotic force $< \hat{y}_n\hat{y}_0 >$ is δ-correlated or decays more rapidly than the change of the distribution function, we can introduce the Markovian approximation, and the stationary distribution $P(x)$ is described as the Gaussian form.

In this paper we study the stationary distribution of the nonlinear system driven by the chaotic force $f(t)$,

$$\dot{x}(t) = (1 + f(t))x(t) - x(t)^2, \tag{3}$$

where $f(t)$ is assumed to be the following form

$$f(t) = \frac{K}{\sqrt{\tau}}g(y_{n+1}) \ \text{ for } \ n\tau < t \leq (n+1)\tau \ \ (n = 0, 1, 2, \ldots). \tag{4}$$

In this paper we consider the following two types as $g(y)$: (a) $g(y_{n+1}) = y_{n+1} - < y_0 >$ and (b) $g(y_{n+1}) = y_{n+1}/ \mid y_{n+1} \mid$.

We have got the following result.

Case(i): $\tau \gg 1$

If the factor $K/\sqrt{\tau}$ is so small that $1 + Kg(y_{n+1})/\sqrt{\tau} > 0$ holds for all n, we get

$$P_{n+1}(x;x_0) \approx \frac{\sqrt{\tau}}{K}\rho(\frac{\sqrt{\tau}}{K}(x-1) + < y_0 >) \quad \text{for type (a)}, \tag{5}$$

$$P_{n+1}(x;x_0) \approx \frac{1}{2}[\delta(x-(1+\frac{K}{\sqrt{\tau}})) + \delta(x-(1-\frac{K}{\sqrt{\tau}}))] \quad \text{for type (b)}. \tag{6}$$

Case(ii): $\tau \ll 1$

If the correlation $< g(y_n)g(y_0) >$ decays very rapidly and the Markovian approximation holds, we can derive the usual Fokker-Planck equation,

$$\frac{P_{n+1}(x;x_0) - P_n(x;x_0)}{\tau} = \frac{\partial}{\partial x}[(-1-s^{-1}+x)xP_n(x;x_0)] + s^{-1}\frac{\partial^2}{\partial x^2}[x^2P_n(x;x_0)], \tag{7}$$

where s^{-1} is defined in terms of the magnitude K and the correlation C,

$$s^{-1} = K^2C, \quad C = \frac{1}{2} < g(y_0)^2 > + \sum_{m=1}^{\infty} < g(y_m)g(y_0) > . \tag{8}$$

The stationary distribution $P(x)$ of (7) is calculated as

$$P(x) = \frac{s^s}{\Gamma(s)}x^{s-1}exp(-sx) = s\Pi(s-1,sx), \tag{9}$$

where $\Pi(k,\mu)$ is the Poisson distribution.

It was shown that the shape of the stationary distribution $P(x)$ depends much on the time interval τ and the magnitude K of the chaotic force. For large τ the stationary distribution has the same form as that of the invariant density of the chaotic force $g(y)$ as in the linear case. On the other hand, for small τ we find from (9) that the stationary distribution $P(x)$ exhibits the drastic change according to s. If $s > 1$, the stationary distribution has a simple peak at $x = (s-1)/s$ and $P(0) = 0$, which means that $P(x)$ has the most probable value at the nontrivial value $x = (s-1)/s$. It should be noted here that the value $x = (s-1)/s$ is smaller than the original stable point value $x = 1$. For large s the distribution (9) can be approximated by the Gaussian form with the mean value $x = 1$. On the other hand, for $s < 1$ the stationary distribution diverges at $x = 0$, and it is a monotonically decreasing function, which means that the point $x = 0$ is the most probable value. The point $x = 0$ was unstable in the case of no chaotic force. In other words ,if the magnitude K or the correlation C is small enough to satisfy the condition: $K < 1/\sqrt{C}$, the stationary distribution has a peak at $x = (s-1)/s$.

A similar result is obtained in the model of the stochastic differential equation with white-Gaussian noise, where the drastic change is called "noise induced transsition".

It should be noted here that the stationary distribution (9) does not depend on the choice $g(y)$.

Reference

1) T.Shimizu and N.Morioka, Physica A **218** (1995) 390-402 ,No3/4.